KTH Matematik

Exam 1, solutions
5B1309 Algebra g.k.
1 Februari, 2006

(1) (2 pts) Consider the following permutation $\sigma \in \mathbb{S}_8$:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 5 & 8 & 1 & 6 & 4 & 7 \end{pmatrix}$$

Is σ even or odd? (motivate your answer!)
$\sigma = (1235)(487) = (15)(13)(12)(47)(48)$, then it is odd.

(2) (4 pts) (motivate your answer!)
(a) Are $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ isomorphic as sets? Both sets have cardinality 4, therefore they are isomorphic.
(b) Consider the sets above with the addition:
- if $[i], [j] \in \mathbb{Z}_4$ the addition is defined modulo 4: $[i] + [j] = [i + j]$;
- if $([i], [j]), ([k], [l]) \in \mathbb{Z}_2 \times \mathbb{Z}_2$ then $([i], [j]) + ([k], [l]) = ([i + k], [j + l])$.

Are $(\mathbb{Z}_4, +)$ and $(\mathbb{Z}_2 \times \mathbb{Z}_2, +)$ isomorphic as groups? No. $(\mathbb{Z}_4, +) = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$. If it were isomorphic to $\mathbb{Z}_4$, then it would contain an element of order 4 and therefore it would be cyclic. But there is no element of order 4, since: $ord(0, 0) = 1, ord(0, 1) = ord(1, 0) = ord(1, 1) = 2$.

(3) (3 pts) (motivate your answer!) List all the subgroups of $\mathbb{Z}_{12}$ and illustrate the list with a diagram. Recall that $\mathbb{Z}_{12}$ is a cyclic group generated by elemnts which are coprime with 12. So:
$\mathbb{Z}_{12} =< 1 >=< 5 >=< 7 >=< 11 >$.
The proper subgroups must be cyclic.

We have that:
$< 2 >= \{0, 2, 4, 6, 8, 10\} =< 10 >$,
$< 3 >= \{0, 3, 6, 9\} =< 3 >$,
$< 4 >= \{0, 4, 8\} =< 8 >$,
$< 6 >= \{0, 6\}$